Leonid Berlyand

(Pennsylvania State University)

Mathematics of Artificial Neural Networks (ANNs).

*We begin by describing a mathematical setting for the image recognition problem using a simple example of the classification of handwritten digits and letters. Here we introduce the notion of the classifier map F(x), which maps an n-dimensional Euclidean space to a discrete set of {1,…, K} of K classes (e.g., number of letters in an alphabet). The key question is how to construct a good approximation F(x, **a**), where **a** is a set of tunable parameters. While approximation theory is an old and well-developed area of mathematics (e.g., well-known Fourier series), the key novel idea was to introduce the parameters **a** by mimicking the human brain, which was a true rebirth of the classical approximation theory. We explain this idea by defining Artificial Neural Networks (ANNs) using simple mathematical concepts such as linear/affine maps and the composition of functions. Then, we explain how ANNs are used in the classification problem for handwritten digits or letters. Here, we focus on explaining the concept of training algorithms using the classical notion of optimization of vector functions. We provide a few examples of how well-known mathematical theories and theorems (e.g., random matrix theory and fixed point theorems) can help in ANNs in Deep Learning. Finally, we briefly mention the 2024 Nobel Prizes in Physics and Chemistry, that recognized foundational work in ANNs and Deep Learning and its critical role in other disciplines.*

Academia das Ciências de Lisboa, 22 de maio de 2025